

One-Bit Decentralized Detection With a Rao Test for Multisensor Fusion

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Abstract—In this letter, we propose the Rao test as a simpler alternative to the generalized likelihood ratio test (GLRT) for multisensor fusion. We consider sensors observing an unknown deterministic parameter with symmetric and unimodal noise. A decision fusion center (DFC) receives quantized sensor observations through error-prone binary symmetric channels and makes a global decision. We analyze the optimal quantizer thresholds and we study the performance of the Rao test in comparison to the GLRT. Also, a theoretical comparison is made and asymptotic performance is derived in a scenario with homogeneous sensors. All the results are confirmed through simulations.

Index Terms—Decentralized detection, Rao test, threshold optimization, wireless sensor networks (WSNs).

I. INTRODUCTION

DECENTRALIZED detection with wireless sensor networks (WSNs) has received close attention by the scientific community over the last decade. Each sensor, rather than sending its observed measurements, typically sends one bit of information about the estimated hypothesis to the decision fusion center (DFC), which makes a global decision. Such an approach is generally employed in order to satisfy stringent constraints on bandwidth and energy. In this context the optimal test (under Bayesian and Neyman-Pearson frameworks) at each sensor is well known to be a one-bit quantization of the local likelihood-ratio test (LRT). Unfortunately in most cases, due to a lack of signal knowledge, it is not possible to compute the local LRT at the generic sensor. Also, even when the sensors can compute their local LRT, the search for local quantization thresholds is well known to be exponentially complex [1], [2]. In such situations the raw measurement is directly quantized into a single bit of information; the DFC is then in charge of solving a composite hypothesis test.

Some simple approaches have been based on the counting rule or channel-aware statistics, which neglect the dependence with respect to (w.r.t.) the unknown signal [3]–[6]. On the other

hand, in some particular scenarios the uniformly most powerful test is independent of the unknown parameters under the alternate hypothesis, which then do not need to be estimated [7]. Nonetheless, typically the fusion rule employed at the DFC is based on the generalized LRT (GLRT). GLRT-based fusion of quantized data was studied in [8], [9], for detecting a source with unknown location and fusing conditionally dependent decisions, respectively. Recently in [10] the GLRT has been used to detect an unknown deterministic signal (in a decentralized fashion with quantized measurements and noisy communication channels of identical quality) and an asymptotically optimal threshold choice for the quantizer has been derived in the non-homogeneous sensor case (i.e. an additive Gaussian observation model with unequal variances).

The contributions of this letter are summarized hereinafter. We study the problem in [10] and we propose the Rao test as a computationally simpler alternative to the GLRT, since it does not require any estimation procedure; its closed form is obtained in the more general case of zero-mean noise with symmetric and unimodal pdf and non-identical bit-error probabilities (BEPs) on the communication channels. Also, we discuss the optimal choice of quantizer threshold for some pdfs of interest. Furthermore, the Rao test is compared to the GLRT through simulations showing that, in addition to sharing the same asymptotic distribution, it achieves practically the same performance for a finite number of sensors. This result becomes in fact theoretical coincidence in a scenario with homogeneous sensors; for the latter scenario a tighter asymptotic distribution of both tests is derived.

The letter is organized as follows: Section II introduces the model; in Section III we derive the Rao test and the corresponding optimal thresholds; in Section IV the GLR and Rao tests are compared analytically in a homogeneous scenario, while in Section V we confirm the results through simulations; in Section VI we draw some conclusions.

II. PROBLEM STATEMENT

The system model is described¹ as follows. We consider a binary hypothesis testing problem in which a collection of sensors $k \in \mathcal{K} \triangleq \{1, \dots, K\}$ collaborate to detect the presence of an

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¹**Notation**—Lower-case bold letters denote vectors, with a_n being the n th element of $\mathbf{a}$; upper-case calligraphic letters, e.g. $\mathcal{A}$, denote finite sets; $\mathbb{E}\{\cdot\}$, $\text{var}\{\cdot\}$ and $(\cdot)^t$ denote expectation, variance and transpose, respectively; $P(\cdot)$ and $p(\cdot)$ are used to denote probability mass functions (pmf) and probability density functions (pdf), respectively, while $P(\cdot|\cdot)$ and $p(\cdot|\cdot)$ their corresponding conditional counterparts; $\mathcal{N}(\mu, \sigma^2)$ denotes a Gaussian pdf with mean μ and variance σ^2 ; χ_k^2 (resp. $\chi_k'^2(\xi)$) denotes a chi-square (resp. a non-central chi-square) pdf with k degrees of freedom (resp. and non-centrality parameter ξ); $\mathcal{U}(a, b)$ denotes a uniform pdf with support $[a, b]$; $\mathcal{L}(\mu, \beta)$ denotes a Laplace pdf with mean μ and scale parameter β ; the symbols $\sim$ and $\hat{\sim}$ mean “distributed as” and “asymptotically distributed as”.

unknown deterministic parameter $\theta \in \mathbb{R}$. The problem can be summarized as follows:

$$\begin{cases} \mathcal{H}_0 : & x_k = w_k, \\ \mathcal{H}_1 : & x_k = h_k\theta + w_k, \end{cases} \quad k \in \mathcal{K}; \quad (1)$$

where $x_k \in \mathbb{R}$ denotes the k th sensor measurement, $h_k \in \mathbb{R}$ is a known observation coefficient and $w_k \in \mathbb{R}$ denotes the noise random variable (RV) with $\mathbb{E}\{w_k\} = 0$ and *unimodal symmetric* pdf,² denoted with $p_{w_k}(\cdot)$. Furthermore, the RVs w_k are assumed mutually independent. It is worth noting that (1) refers to a *two-sided test* [11], where $\{\mathcal{H}_0, \mathcal{H}_1\}$ corresponds to $\{\theta = \theta_0, \theta \neq \theta_0\}$ (in our case $\theta_0 = 0$).

Also, to meet stringent bandwidth and power budgets in WSNs, the k th sensor quantizes³ x_k into one bit of information, that is $b_k \triangleq u(x_k - \tau_k)$, $k \in \mathcal{K}$, with $u(\cdot)$ denoting the Heaviside (unit) step function and τ_k the quantizer threshold. The quantized measurement b_k is sent over a binary symmetric channel (BSC) and the DFC observes a (communication) error-prone y_k , that is $y_k = b_k$ with probability $1 - P_{e,k}$ and $y_k = 1 - b_k$ with probability $P_{e,k}$, which we collect as $\mathbf{y} \triangleq [y_1 \cdots y_K]^t$. Here $P_{e,k}$ denotes the BEP of k th link. The problem here is the derivation of a (computationally) simple test on the basis of $\mathbf{y}$ and the quantizer design for each sensor (i.e. an optimized τ_k , $k \in \mathcal{K}$).

III. RAO TEST

A. Test Derivation

A common approach to detection in composite hypothesis testing problems is given by the GLRT, which has been derived and studied in [10] for the model under investigation and whose expression is:

$$\left\{ \Lambda_G \triangleq 2 \cdot \ln \left[\frac{P(\mathbf{y}; \hat{\theta}_1)}{P(\mathbf{y}; \theta_0)} \right] \right\} \begin{matrix} \mathcal{H} = \mathcal{H}_1 \\ \geq \gamma \\ \mathcal{H} = \mathcal{H}_0 \end{matrix} \quad (2)$$

where $P(\mathbf{y}; \theta)$ denotes the likelihood as a function of θ , $\hat{\theta}_1$ is the *maximum likelihood (ML) estimate* under $\mathcal{H}_1$ (i.e. $\hat{\theta}_1 \triangleq \arg \max_{\theta} P(\mathbf{y}; \theta)$) and γ is the threshold. It is clear from (2) that Λ_G requires the solution to an optimization problem; this increases the computational complexity of its implementation. However, in the special case $w_k \sim \mathcal{N}(0, \sigma_k^2)$ it was shown in [12] that ML estimation is a convex problem and thus it can be efficiently solved with local-optimization routines. Unfortun-

²Noteworthy examples of such pdfs are the Gaussian, Laplace, Cauchy and generalized Gaussian distributions with zero mean [11].

³We restrict our attention to deterministic quantizers for simplicity; an alternative is the use of stochastic quantizers, however their analysis falls beyond the scope of this letter.

nately a closed form for $\hat{\theta}_1$ is not available even under such an assumption.

As such, we pursue the derivation of the Rao test [11], which for the scalar case ($\theta \in \mathbb{R}$) is given implicitly as:

$$\left\{ \Lambda_R \triangleq \frac{\left(\frac{\partial \ln P(\mathbf{y}; \theta)}{\partial \theta} \Big|_{\theta=\theta_0} \right)^2}{I(\theta_0)} \right\} \begin{matrix} \mathcal{H} = \mathcal{H}_1 \\ \geq \gamma \\ \mathcal{H} = \mathcal{H}_0 \end{matrix} \quad (3)$$

where $I(\theta_0)$ is the *Fisher information (FI)*, i.e. $I(\theta) \triangleq \mathbb{E}\{(\partial \ln P(\mathbf{y}; \theta) / \partial \theta)^2\}$ evaluated at θ_0 . The motivation of our choice is the extreme simplicity of the test implementation (since $\hat{\theta}_1$ is not required, cf. (3)), but with the same weak-signal asymptotic performance as the GLRT [11].

In order to obtain Λ_R explicitly, we expand $\ln[P(\mathbf{y}; \theta)]$ as:

$$\begin{aligned} \ln[P(\mathbf{y}; \theta)] &= \sum_{k=1}^K \ln[P(y_k; \theta)] \\ &= \sum_{k=1}^K \{y_k \cdot \ln[(1 - P_{e,k})\alpha_k(\theta) + P_{e,k}(1 - \alpha_k(\theta))] \\ &\quad + (1 - y_k) \\ &\quad \cdot \ln[(1 - P_{e,k})(1 - \alpha_k(\theta)) + P_{e,k}\alpha_k(\theta)]\} \end{aligned} \quad (4)$$

where $\alpha_k(\theta) \triangleq F_{w_k}(\tau_k - h_k\theta)$, with $F_{w_k}(\cdot)$ denoting the complementary cumulative distribution function of w_k . On the other hand, $I(\theta)$ is given in closed form [10] as:

$$\begin{aligned} I(\theta) &= \sum_{k=1}^K \left\{ \frac{(1 - 2P_{e,k})^2 \cdot h_k^2 \cdot p_{w_k}^2(\tau_k - h_k\theta)}{P_{e,k} + (1 - 2P_{e,k}) \cdot F_{w_k}(\tau_k - h_k\theta)} \right. \\ &\quad \times \left. \frac{1}{[1 - P_{e,k} - (1 - 2P_{e,k}) \cdot F_{w_k}(\tau_k - h_k\theta)]} \right\}. \end{aligned} \quad (5)$$

Combining (4) and (5) we obtain Λ_R in closed form, as shown in (6) at the bottom of the page. It is apparent that Λ_R (as well as Λ_G) is a function of τ_k , $k \in \mathcal{K}$, which can be optimized to achieve (asymptotically) optimal performance. (See equation at bottom of page)

B. Quantizer Design With Asymptotic Performance Analysis

We know from theory that Λ_R (as well as Λ_G), is asymptotically (when the signal is weak⁴) distributed as follows [11]:

$$\Lambda_R \stackrel{a}{\sim} \begin{cases} \chi_{1^2}^2 & \text{under } \mathcal{H}_0 \\ \chi_{1^2}^2(\lambda_Q) & \text{under } \mathcal{H}_1 \end{cases} \quad (7)$$

where the non-centrality parameter λ_Q is given by:

$$\lambda_Q \triangleq (\theta_1 - \theta_0)^2 I(\theta_0) \quad (8)$$

⁴That is $|\theta_1 - \theta_0| = c/\sqrt{K}$ for some constant $c > 0$ [11].

$$\Lambda_R = \left(\sum_{k=1}^K \frac{(1 - 2 \cdot P_{e,k}) \cdot h_k \cdot p_{w_k}(\tau_k) \cdot [2y_k - 1]}{(1 - P_{e,k}) \cdot F_{w_k}(\tau_k)^{y_k} \cdot [1 - F_{w_k}(\tau_k)]^{1-y_k} + P_{e,k} \cdot F_{w_k}(\tau_k)^{1-y_k} \cdot [1 - F_{w_k}(\tau_k)]^{y_k}} \right)^2 \times (I(\theta = 0))^{-1} \quad (6)$$

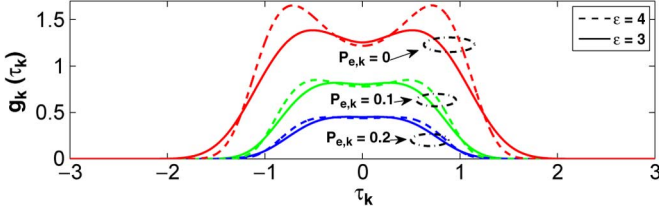


Fig. 1. Effect of $P_{e,k}$ on $g_k(\tau_k)$ when $p_{w_k}(\tau_k) = (\epsilon/2\alpha\Gamma(1/\epsilon)) \exp[-(|\tau_k|/\alpha)^\epsilon]$; $\alpha = 1$, $\epsilon \in \{3, 4\}$ and $P_{e,k} \in \{0, 0.1, 0.2\}$.

with θ_1 being the true value under $\mathcal{H}_1$. Clearly the larger λ_Q , the better the GLRT and Rao tests will perform. Also, as shown in [10], λ_Q is a function of τ_k , $k \in \mathcal{K}$; therefore we choose τ_k , $k \in \mathcal{K}$, in order to maximize λ_Q , that is

$$\arg \max_{\{\tau_1, \dots, \tau_K\}} \left\{ \lambda_Q = \theta^2 \sum_{k=1}^K \left[\frac{(1 - 2P_{e,k})^2 \cdot h_k^2 \cdot p_{w_k}^2(\tau_k)}{P_{e,k} + (1 - 2P_{e,k}) \cdot F_{w_k}(\tau_k)} \right] \right\}, \quad (9)$$

which can be decoupled into the following set of K independent threshold design problems:

$$\arg \max_{\tau_k} \left\{ g_k(\tau_k) \triangleq \frac{p_{w_k}^2(\tau_k)}{\Delta_k + F_{w_k}(\tau_k) \cdot [1 - F_{w_k}(\tau_k)]} \right\} \quad (10)$$

where $\Delta_k \triangleq [P_{e,k} \cdot (1 - P_{e,k})]/(1 - 2P_{e,k})^2$. It is known from quantized estimation literature [13], [14] that many unimodal and symmetric $p_{w_k}(\cdot)$'s with $\mathbb{E}\{w_k\} = 0$ lead to $\tau_k^* \triangleq \arg \max_{\tau_k} g_k(\tau_k) = 0$ (independent of Δ_k); such examples are the Gaussian, Laplace, Cauchy and the widely used generalized normal distribution, that is $p_{w_k}(\tau_k) = (\epsilon/2\alpha\Gamma(1/\epsilon)) \exp[-(|\tau_k|/\alpha)^\epsilon]$, only when $0 \leq \epsilon \leq 2$; on the other hand when $\epsilon > 2$, $g_k(\tau_k)$ becomes bimodal (since it is symmetric) as shown in Fig. 1. However the effect of a non-ideal BSC smoothes the gain achieved by τ_k^* and thus $\tau_k = 0$ is still a good (sub-optimal) choice. Substituting $\tau_k = 0$, $k \in \mathcal{K}$, in (6), leads to the following simplified expression for threshold-optimized Rao test (denoted with Λ_R^*):

$$\Lambda_R^* = \frac{4 \cdot \left[\sum_{k=1}^K (1 - 2P_{e,k}) \cdot p_{w_k}(0) \cdot h_k \cdot (y_k - \frac{1}{2}) \right]^2}{\sum_{k=1}^K (1 - 2P_{e,k})^2 \cdot p_{w_k}^2(0) \cdot h_k^2} \quad (11)$$

which is considerably simpler than the GLRT, as it obviates solution of an optimization problem (which depends on $p_{w_k}(\cdot)$). Furthermore, the corresponding optimized non-centrality parameter, denoted with λ_Q^* , is given by:

$$\lambda_Q^* = 4\theta^2 \cdot \sum_{k=1}^K [(1 - 2P_{e,k})^2 \cdot p_{w_k}^2(0) \cdot h_k^2] \quad (12)$$

Remarks: In the case of BSCs of the same quality (i.e. $P_{e,k} = P_e$, $k \in \mathcal{K}$) we simply get $\lambda_Q^* = (1 - 2P_e)^2 \cdot \lambda_{Q_0}^*$, where $\lambda_{Q_0}^* \triangleq 4\theta^2 \cdot \sum_{k=1}^K [p_{w_k}^2(0) \cdot h_k^2]$ represents λ_Q^* in the ideal BSC case ($P_{e,k} = 0$, $k \in \mathcal{K}$). This result generalizes the one in [10], by stating that the loss due to non-ideal communications is asymptotically independent of $p_{w_k}(\cdot)$, $k \in \mathcal{K}$.

IV. COMPARISON IN HOMOGENEOUS SCENARIO

In this section we study the simplified scenario $h_k = h$, $p_{w_k}(\cdot) = p_w(\cdot)$, $P_{e,k} = P_e$, $k \in \mathcal{K}$, to get an intuitive interpretation of the two threshold-optimized tests ($\tau_k^* = 0$). Based on these assumptions, the statistics in (2) and (11) reduce to:

$$\Lambda_G^* = 2K \cdot \left[\hat{\rho} \ln \left(\frac{\hat{\rho}}{\rho_0} \right) + (1 - \hat{\rho}) \ln \left(\frac{1 - \hat{\rho}}{1 - \rho_0} \right) \right] \quad (13)$$

$$= 2K \cdot D_{\text{KL}} \left(\hat{P}(y_k) \parallel P(y_k; \theta_0) \right) \quad (14)$$

$$\Lambda_R^* = 4K \cdot [\hat{\rho} - \rho_0]^2 \quad (15)$$

$$= 4K \cdot \left[D_{\text{TVD}} \left(\hat{P}(y_k) \parallel P(y_k; \theta_0) \right) \right]^2 \quad (16)$$

where $\Lambda_G^* \triangleq \Lambda_G(\tau_k = 0)$, $\hat{\rho} \triangleq \sum_{k=1}^K y_k / K$ and $\rho_0 \triangleq 1/2$. Here $\hat{P}(y_k)$ represents the empirical distribution of the i.i.d. binary source $\{y_1, \dots, y_K\}$ and $D_{\text{KL}}(\cdot \parallel \cdot)$ and $D_{\text{TVD}}(\cdot \parallel \cdot)$ denote the *Kullback-Leibler* (KL) and *total variation distance* (TVD) divergences, respectively [15]. It is worth noticing that in (14) we exploited the closed form of $\hat{\theta}_1 = -(1/h)F_w^{-1}((\hat{\rho} - P_e)/(1 - 2P_e))$ (see [12] for a similar result).

Exploiting KL⁵ and TVD divergences properties it can be shown that both (14) and (16) are monotone (increasing) functions of $|\hat{\rho} - \rho_0|$ and therefore *represent equivalent tests* in a homogeneous sensor scenario, meaning their performances coincide also for a finite number of sensors.

Finally, we derive a tighter asymptotic form of the conditional pdf (*not requiring* the weak-signal assumption) of both the tests in this scenario with the help of the central limit theorem (CLT) [15]. Without loss of generality we focus hereinafter on Λ_R^* (since Λ_G^* has the same performance). For this purpose, we define the RV $\xi \triangleq \sum_{k=1}^K (2y_k - 1)/\sqrt{K}$ and we consider the asymptotic form of $p_\xi(\cdot|\mathcal{H}_i)$, $i \in \{0, 1\}$, which according to the CLT is given as $K \rightarrow +\infty$ by:

$$\xi|\mathcal{H}_0 \stackrel{a}{\sim} \mathcal{N}(0, 1) \quad \xi|\mathcal{H}_1 \stackrel{a}{\sim} \mathcal{N}(\sqrt{K}\tilde{\mu}_1, \tilde{\sigma}_1^2) \quad (17)$$

where $\tilde{\mu}_1 \triangleq (1 - 2P_e)(2\rho_1 - 1)$, $\tilde{\sigma}_1^2 \triangleq 4 \cdot [1 + P_e(2\rho_1 - 1) - \rho_1] \cdot [\rho_1 + (1 - 2\rho_1)P_e]$ and $\rho_1 \triangleq F_w(-h\theta)$. From inspection of (15), it can be verified that $\Lambda_R^* = \xi^2$ holds, which can be exploited to obtain closed form performance expressions.

V. NUMERICAL RESULTS

In this section we compare the Rao test to the GLRT. We evaluate the performance in terms of system false alarm and detection probabilities, defined as $P_{F_0} \triangleq \Pr\{\Lambda > \gamma|\mathcal{H}_0\}$ and $P_{D_0} \triangleq \Pr\{\Lambda > \gamma|\mathcal{H}_1\}$, respectively, where Λ is the statistic employed at the DFC. We also define the k th sensor observation signal-to-noise ratio (SNR) as $\Gamma_k \triangleq (h_k^2\theta^2/\mathbb{E}\{w_k^2\})$.

In Fig. 2 we illustrate P_{D_0} vs P_{F_0} in a WSN with $K = 5$ sensors where $\theta = 1$, $h_k \sim U(0, a)$, $k \in \mathcal{K}$ (but known at the DFC), and two noise pdfs: (i) $w_k \sim \mathcal{N}(0, \sigma_k^2)$ and (ii) $w_k \sim \mathcal{L}(0, \beta_k)$, such that $\mathbb{E}\{w_k^2\} = 1$. We consider four combinations corresponding to $P_{e,k} = P_e \in \{0, 0.2\}$ and $\bar{\Gamma}_{dB} \in \{0, 10\}$, where we have denoted $\bar{\Gamma} \triangleq \mathbb{E}\{\Gamma_k\}$ (in our case $\bar{\Gamma} = (a^2\theta^2)/3 \cdot \mathbb{E}\{w_k^2\}$) as the average observation SNR. The figures are based on 10^5 Monte Carlo runs. First, it is apparent that the

⁵Since it is increasing when $\hat{\rho} > \rho_0$ and symmetric around ρ_0 .

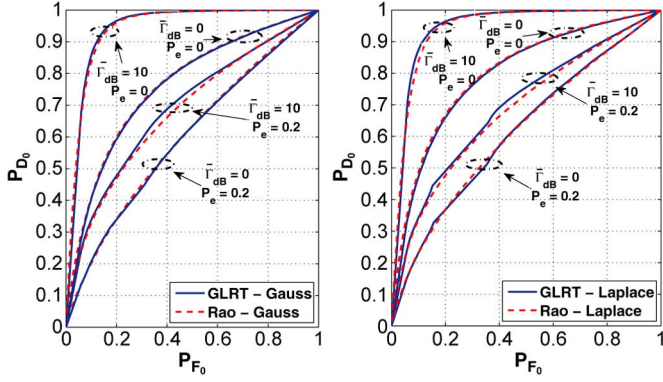


Fig. 2. P_{D0} vs P_{F0} ; WSN with $K = 5$ sensors, $h_k \sim \mathcal{U}(0, a)$, $\theta = 1$, $\mathbb{E}\{w_k^2\} = 1$ for Gaussian and Laplace noise; $P_e \in \{0, 0.2\}$, $\bar{\Gamma}_{dB} \in \{0, 10\}$.

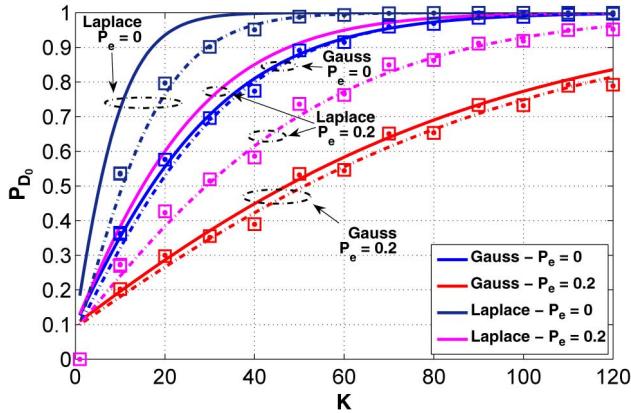


Fig. 3. P_{D0} vs K ; $P_{F0} = 0.1$. Setup: $\theta = 0.5$; $h_k = 1$, $\mathbb{E}\{w_k^2\} = 1$ ($(\Gamma_k)_{dB} \approx -6$), $P_{e,k} \in \{0, 0.2\}$, homogeneous scenario. Square ($\square$) and bullet ($\bullet$) markers refer to GLRT and Rao test, respectively; solid and dash-dot lines refer to weak-signal and CLT-based asymptotic pdfs, respectively.

performances of the GLR and the Rao tests are practically the same for all the considered scenarios; however the implementation of the Rao test is much simpler than that of the GLRT. Also, the difference in performances under Laplacian and Gaussian noises is significant only at $\bar{\Gamma}_{dB} = 0$, while at $\bar{\Gamma}_{dB} = 10$ the curves almost overlap. This is explained since when $\bar{\Gamma}$ is low the signal is more concentrated around zero. Then the imbalance in the binary pmf observed at the output of each quantizer is higher when $w_k \sim \mathcal{L}(0, \beta_k)$.

In Fig. 3 we show P_{D0} as a function of K , assuming $P_{F0} = 0.1$. We consider $\theta = 0.5$, $h_k = 1$ and two noise pdfs: (i) $w_k \sim \mathcal{N}(0, \sigma_k^2)$ and (ii) $w_k \sim \mathcal{L}(0, \beta_k)$, such that $\mathbb{E}\{w_k^2\} = 1$ (thus $(\Gamma_k)_{dB} \approx -6$), $k \in \mathcal{K}$. Also, we consider $P_e \in \{0, 0.2\}$, thus determining a homogeneous scenario. First, Monte Carlo simulations confirm the theoretical coincidence between the Rao test (bullet markers) and the GLRT (square markers). Secondly, it is apparent that the CLT-based performance expressions (dash-dot) are as accurate as those based on the weak-signal assumption (solid lines) for Gaussian noise, while in the Laplacian case the weak-signal distribution is far from being representative of the distribution. Interestingly, when $P_e = 0$, λ_{Q0}^* in the Laplacian case coincides with the non-centrality parameter achieved by a GLRT (or Rao test) based on the raw x_k , $k \in \mathcal{K}$, given by $\lambda_{UQ} = \theta^2 \sum_{k=1}^K (h_k^2 / \beta_k^2)$, that is (12) *does not predict the loss due to quantization*. On the other hand, by exploiting the CLT-based performance in (17), we can compare λ_{UQ} with the modified deflection coefficient of the asymptotic problem

given by (17) $d_Q \triangleq K \tilde{\mu}_1^2 / \tilde{\sigma}_1^2$, which for the Laplacian noise is given by $d_Q = (K \theta^2 (h^2 / \beta^2) / [1 - \exp(-|h\theta|/\beta)]^2) \cdot [1 - (1 - \exp(-|h\theta|/\beta))^2]$; thus for this problem we have $\lambda_{UQ} / d_Q \approx 1.45$, which predicts the performance loss well.

VI. CONCLUSIONS

We studied the Rao test for decentralized detection with an unknown deterministic signal as an attractive alternative to GLRT for a general model with quantized measurements, zero-mean, unimodal and symmetric noise (pdf), non-ideal and non-identical BSCs. The asymptotically optimal sensor thresholds were shown to be zero for many pdfs of interest and a fair choice in other scenarios; this result was exploited to simplify further the Rao test. Also, it was shown through simulations that the Rao test, in addition to being asymptotically equivalent to the GLRT, achieves practically the same performance in the finite number of sensors case; for the case of homogeneous sensors a theoretical coincidence of the two tests was established. In such a scenario a general asymptotic performance were derived based on the CLT and not requiring the weak-signal assumption. These latter were shown to be crucial in performance analysis with peaked noise pdfs.

REFERENCES

- [1] J. N. Tsitsiklis, "Decentralized detection," *Adv. Statist. Signal Process.*, vol. 2, no. 2, pp. 297–344, 1993.
- [2] R. Viswanathan and P. K. Varshney, "Distributed detection with multiple sensors—Part I: Fundamentals," *Proc. IEEE*, vol. 85, no. 1, pp. 54–63, Jan. 1997.
- [3] V. A. Aalo and R. Viswanathan, "Multilevel quantisation and fusion scheme for the decentralised detection of an unknown signal," *Proc. Inst. Elect. Eng., Radar, Sonar Navig.*, vol. 141, no. 1, pp. 37–44, Feb. 1994.
- [4] B. Chen, R. Jiang, T. Kasetkasem, and P. K. Varshney, "Channel aware decision fusion in wireless sensor networks," *IEEE Trans. Signal Process.*, vol. 52, no. 12, pp. 3454–3458, Dec. 2004.
- [5] R. Niu and P. K. Varshney, "Performance analysis of distributed detection in a random sensor field," *IEEE Trans. Signal Process.*, vol. 56, no. 1, pp. 339–349, Jan. 2008.
- [6] D. Ciuonzo, G. Romano, and P. Salvo Rossi, "Channel-aware decision fusion in distributed MIMO wireless sensor networks: Decode-and-fuse vs. decode-then-fuse," *IEEE Trans. Wireless Commun.*, vol. 11, no. 8, pp. 2976–2985, Aug. 2012.
- [7] D. Ciuonzo, G. Romano, and P. Salvo Rossi, "Optimality of received energy in decision fusion over Rayleigh fading diversity MAC with non-identical sensors," *IEEE Trans. Signal Process.*, vol. 61, no. 1, pp. 22–27, Jan. 2013.
- [8] R. Niu and P. K. Varshney, "Joint detection and localization in sensor networks based on local decisions," in *Fortieth Asilomar Conf. Signals, Systems and Computers*, 2006, pp. 525–529.
- [9] S. G. Iyengar, R. Niu, and P. K. Varshney, "Fusing dependent decisions for hypothesis testing with heterogeneous sensors," *IEEE Trans. Signal Process.*, vol. 60, no. 9, pp. 4888–4897, Sep. 2012.
- [10] J. Fang, Y. Liu, H. Li, and S. Li, "One-bit quantizer design for multisensor GLRT fusion," *IEEE Signal Process. Lett.*, vol. 20, no. 3, pp. 257–260, Mar. 2013.
- [11] S. M. Kay, *Fundamentals of Statistical Signal Processing, Volume 2: Detection Theory*. Upper Saddle River, NJ, USA: Prentice-Hall PTR, Jan. 1998.
- [12] A. Ribeiro and G. B. Giannakis, "Bandwidth-constrained distributed estimation for wireless sensor networks—Part I: Gaussian case," *IEEE Trans. Signal Process.*, vol. 54, no. 3, pp. 1131–1143, Mar. 2006.
- [13] H. C. Papadopoulos, G. W. Wornell, and A. V. Oppenheim, "Sequential signal encoding from noisy measurements using quantizers with dynamic bias control," *IEEE Trans. Inf. Theory*, vol. 47, no. 3, pp. 978–1002, Mar. 2001.
- [14] D. Rousseau, G. V. Anand, and F. Chapeau-Blondeau, "Nonlinear estimation from quantized signals: Quantizer optimization and stochastic resonance," in *Proc. 3rd Int. Symp. Physics in Signal and Image Processing*, 2003, pp. 89–92.
- [15] T. M. Cover and J. A. Thomas, *Elements of Information Theory*. Hoboken, NJ, USA: Wiley-Interscience, 2006.